

## Simulations of pulsar wind formation

Anatoly Spitkovsky and Jonathan Arons

*Astronomy Dept., University of California, Berkeley, CA 94720, USA*

**Abstract.** We present initial results of the first self-consistent numerical model of the outer magnetosphere of a pulsar. By using the relativistic “particle-in-cell” method with special boundary conditions to represent plasma dynamics in 3D, we are able to follow magnetospheric plasma through the light cylinder into the wind zone for arbitrary magnetic inclination angles. For aligned rotators we confirm the “disk-dome” charge-separated structure of the magnetosphere and find that this configuration is unstable to a 3D nonaxisymmetric diocotron instability. This instability allows plasma to move across the field lines and approach the corotating Goldreich-Julian solution within several rotation periods. For oblique rotators formation of the spiral “striped wind” in the equatorial direction is demonstrated and the acceleration of the wind and its magnetization is discussed. We find that the wind properties vary with stellar latitude; however, whether injection conditions at the pulsar are responsible for the observed jet-equator geometry of Crab and Vela is currently under investigation. We also comment on the electrodynamics of the simulated magnetospheres, their current closure, and future simulations.

### 1. Introduction

Most of the pulsar wind models assume a relativistic MHD-type flow with various degrees of symmetry (e.g., Kennel and Coroniti 1984, Begelman and Li 1992), and are reasonably successful in explaining the overall morphology and energetics of the surrounding plerion. However, they run into various difficulties under attempts to extrapolate the solution back towards the pulsar. The most famous of these is the problem of the magnetization of the wind ( $\sigma$ -paradox), which is the contradiction between the low ratio of magnetic to kinetic energy ( $\sigma \sim 10^{-3}$ ) at the wind-nebula interface inferred from observations, *expected* strong magnetization near the pulsar ( $\sigma \gg 1$ ), and the conservation of  $\sigma$  in an ideal MHD flow. Recent observations of the Crab and Vela with Chandra and HST have underscored the importance of understanding the origins of the pulsar wind. Observations of pulsar jets and orthogonal equatorial outflows suggest that rotation of the neutron star is imprinted in the wind structure, possibly at a very early stage. How and where acceleration and collimation of the wind occurs can be reliably studied only by constructing a wind model that begins from the pulsar itself. The main difficulty in the way of theoretical progress on this front is the lack of intuition about the phenomena occurring near a pulsar. For a general case of an oblique rotator, and, as we show in this paper, even for an

aligned rotator, the problem is intrinsically three dimensional, nonaxisymmetric, and time-dependent, or in other words “too complicated”. With observations unlikely to resolve light cylinder scales ( $\sim 10^8$  cm) any time soon, our only resort is numerical simulations. In this paper we describe our effort to simulate the formation of pulsar winds from the first principles. In particular, we study the behavior of plasma that is either emitted from or injected in the vicinity of a strongly magnetized rotating conducting sphere with arbitrary inclination angles between the magnetic and rotational axes. Despite its implied simplicity, this problem has very interesting solutions that may at first appear strange and unusual. Hence, in describing these preliminary results our emphasis is more on developing intuition, rather than on direct relevance to pulsars and observational implications. Such issues as well as more realistic models are currently being investigated. The paper is organized as follows: in § 2 we describe the particle-in-cell method used in our simulations and the setup of the problem, in § 3 and § 4 we describe numerical experiments with aligned and oblique rotators, and in § 5 we conclude and discuss further work.

## 2. Numerical method

We are interested in studying the outer magnetosphere of the pulsar – the region around the light cylinder ( $R_{LC} = c/\Omega$ , where  $\Omega$  is the angular frequency of the star). The possible physical conditions in this region put stringent demands on a successful simulation method. Previous theoretical investigations suggest that the method should be able to handle relativistic flows, vacuum gaps, charge-separated configurations, space-charge limited flows and counterstreaming flows, as well as MHD flow when it is applicable. For oblique rotators the light cylinder is a region of emission of large-amplitude electromagnetic waves, so strong wave-plasma interaction should be also addressable by the method. The requirement of simulating counterstreaming, i.e. flows with multivalued velocity, dispenses with the option of using fluid grid-based schemes. As our method we therefore chose a particle-based “Particle-In-Cell” (PIC) algorithm. It is a fully relativistic multidimensional method for self-consistent solution of Maxwell’s equations in the presence of plasma. The underlying philosophy of PIC is the following: a plasma is represented as a collection of macroparticles which carry charge and mass; the macroparticles are moved by leap-frog integration of the relativistic equations of motion with Lorentz force; the currents and charges associated with the macroparticles are deposited on an Eulerian grid, on which Maxwell’s equations are also discretized; the fields are updated (with particle currents and charges as sources) and extrapolated to the position of the macroparticles to contribute to the Lorentz force for the next time step.

The main advantage of using the PIC method for the pulsar wind problem is that one does not have to impose restrictions on plasma behavior in advance. The method can easily handle charge-separated flows and counterstreaming. It is intrinsically a kinetic model, so particle acceleration can be studied. The ideal MHD condition  $\vec{E} + (\vec{v}/c) \times \vec{B} = 0$  can be achieved in the appropriate regime, and has been observed in our tests. The disadvantage of using such a fundamental plasma model is that multiple spatial and time scales need to be

resolved, which introduces constraints on the time step and spatial resolution (Birdsall and Langdon 1991).

We have modified the publically available 3D relativistic PIC code TRISTAN (Buneman 1993) to simulate plasma dynamics in the magnetosphere of a magnetized conducting rotating sphere. As is well known, electric fields are induced on the surface of such a body, and should serve as the boundary conditions for the field solve. In order to avoid the asymmetries introduced from the discretization of a rotating sphere on a Cartesian grid, we used the linearity of Maxwell's equations to represent the field  $\vec{E}_{total}$ , which is used to advance the plasma particles, as a superposition:  $\vec{E}_{total} = \vec{E}_{vacuum} + \vec{E}_{plasma}$ , where  $\vec{E}_{vacuum}$  is the field of the “vacuum rotator”, given by an analytic formula (Deutsch, 1955), and  $\vec{E}_{plasma}$  is the “plasma” field computed from the currents deposited by macroparticles. The same decomposition is done for the magnetic field. We note that this is not a small amplitude expansion, this decomposition is exact, and two components can be of equal magnitude. A spherical surface at the center of the domain acts a source and sink of macroparticles. This surface can represent the neutron star or a rigidly corotating inner magnetosphere extending to a large fraction of the light cylinder (presently the radius of the sphere  $a > 0.1R_{LC}$ ). The outer walls of the computational domain in the simulation have radiation boundary condition for the fields and particles.

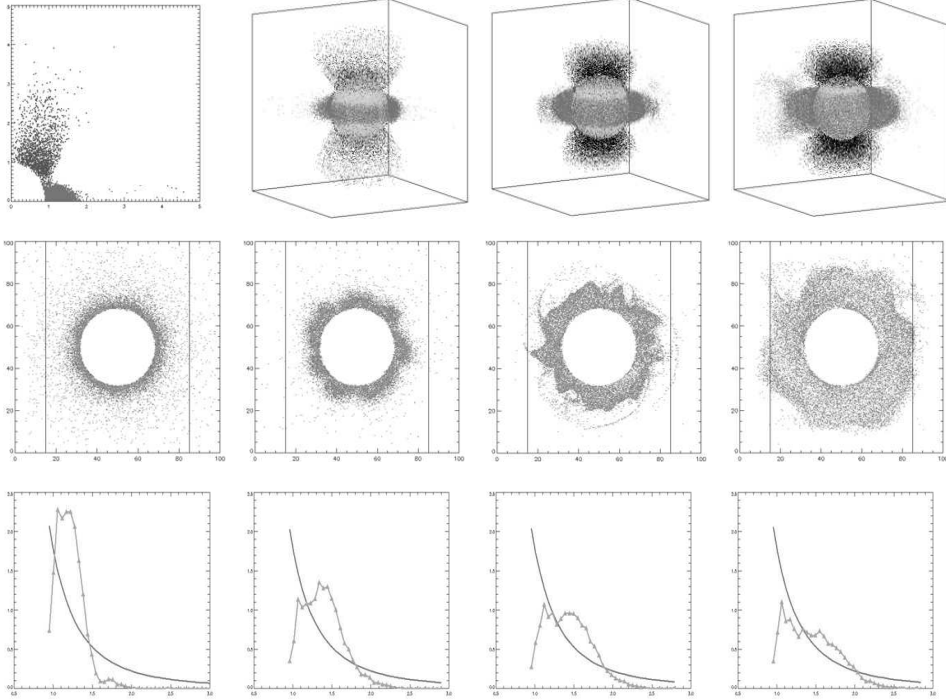


Figure 1. *Top row:* Azimuthally averaged disk-dome configuration, and the 3D view of the magnetosphere filling. *Middle row:* development of the diocotron instability in the equatorial plane. Vertical lines represent the extent of the light cylinder for corotating plasma. *Bottom row:* Evolution of plasma density in the equatorial plane with the corresponding GJ solution (solid line).

### 3. Aligned rotator

We begin by considering the case of magnetic axis being parallel to the rotation axis. There are no time-varying magnetic fields in this case, and the plasma in the magnetosphere will know that the star is rotating only through the electric fields of the unipolar inductor. Therefore, it is interesting to see whether the magnetosphere can fill and reach corotation given a source of plasma. It is natural to expect the plasma to be provided from the surface of the star, as the induced electric field is likely to overwhelm any conceivable work function. At the start of the simulation the rotating star has an induced quadrupolar surface charge and a central monopole giving rise to the following electric field components in spherical coordinates (e.g., Michel and Li 1999):  $E_r = \frac{2}{3}\phi_0\frac{a}{r^2} + \phi_0\frac{a^3}{r^4}(1 - 3\cos^2\theta)$ ,  $E_\theta = -\phi_0\frac{a^3}{r^4}\sin 2\theta$ , where  $\phi_0 = \Omega B_0 a^2/c$ , with  $B_0$  magnetic field at the pole. Plasma is introduced as electron-positron pairs at the stellar surface at each time step. The surface field extracts one sign of charge while driving the other sign into the star where it is arrested. This is equivalent to the surface charge being torn off the star, eventually achieving a configuration where the surface charge due to the vacuum fields is equal and opposite to the surface charge induced by extracting the plasma particles, and the  $\mathbf{E}_{total} \cdot \mathbf{B} = 0$  is satisfied on the surface (strictly speaking, however, the charge left behind after extracting the plasma is not distributed over the star; in this sense the star in the simulation is a dielectric, rather than a conductor; see discussion in §5). The vacuum fields are such that electrons are extracted over the poles of the star and positrons in the equatorial region. This creates a charge-separated plasma with a vacuum region in between. The quadrupolar field of this plasma eventually counteracts the induced vacuum field at the surface, emission of new charges is slowed down and the magnetosphere reaches a quasi-stable disk-dome configuration as shown in Fig. 1. So far our results are in agreement with the previous axisymmetric simulations using a different method (Krause-Polstorff and Michel 1984 (KPM1) and 1985 (KPM2); Smith et al 2001 (SMT)). However, the 3D nature of our simulation allows us to address new physics.

The disk-dome configuration of Fig. 1 is not in rigid corotation with the star. The equatorial disk consists of charges of only one sign and has both velocity shear due to  $E \times B$  drift in the total vacuum+plasma field and a density gradient. These conditions are favorable to the growth of the non-neutral plasma analog of the Kelvin-Helmholtz instability – the diocotron instability. This instability proceeds by amplification of the azimuthal perturbations in the charge density, and is illustrated in Fig. 1. The instability feeds on the energy of shear in the flow, and in the process can move the plasma across the field lines (hence the expanding radius of the disk in fig. 1). Indeed, an azimuthal perturbation in charge density has a nonzero  $E_\theta$  component of the field, which results in a radial component of the  $E \times B$  drift.

Such radial drifts represent the solution to the paradox of Holloway (1973), who noted that there are certain regions in the Goldreich-Julian (1969, GJ) solution that, if evacuated, would not be directly replenished from the star with the correct sign of charge. What probably happens in reality is that regions that are in direct contact with the star will lose corotation and transfer charge across the field lines due to the growing azimuthal perturbations in the charge density. They would then be replenished directly from the surface. Our simulations

show similar behavior. The bottom panel of Fig. 1 displays the evolution of charge density in the equatorial plane as a function of radius. As the instability develops, the density tends to the Goldreich-Julian value. The agreement is not perfect, however: the GJ solution implies an infinite extent of the charge. In our case, the magnetosphere filled only up to the light cylinder, with occasional filaments of charge being torn off after moving past  $R_{LC}$ . In this sense, we did not get a self-consistent wind with the aligned model. Part of the reason is the absence of the global axial current that would significantly alter the magnetic topology. We believe this can be modified with the more realistic boundary conditions (see §5). However, even now it is clear that the GJ solution, corotation and the filling of the magnetosphere with plasma are likely an end result of a robust dynamical process, which is intrinsically three-dimensional in nature.

#### 4. Oblique rotator

With the three dimensional problem set up as described above, studying oblique rotator amounts to just dialing the inclination angle in the formula for the Deutsch field. For the test cases considered here we also modified the plasma injection condition. At every step we injected cold uniform pair plasma within a sphere of radius  $R_{LC}$ , in a deliberately crude imitation of the pair formation process. If the injected plasma density is larger than  $\sim \rho_{GJ}$ , the plasma has a significant dynamical effect on the field, and after an initial transient the system achieves a steady state (number of particles in the domain is constant). The flow consists of two main regions: the closed, quasi-corotating magnetosphere and the outflow (Fig. 2a). The first is the region of the size of  $R_{LC}$  encompassing the closed magnetic field lines. Here the initially neutral plasma is separated with excess positive charge around the equator and excess negative charge around the poles, similar to the GJ solution. The near-corotation is accomplished due to this charge separation and due to the induced electric field that arises from time variation of the magnetic field. Near the light cylinder the plasma is accelerated due to the radiation pressure of the large amplitude electromagnetic wave and forms an outflow. An interesting feature of this wind is that it is formed at the edge of the *closed* zone, and is mostly in the sector around the rotational equator, where it has the characteristic spiral wave pattern (Fig. 2b). Other regions of outflow are also possible. In Fig. 2a we introduced an additional plasma source in the polar flux tube for a  $20^\circ$  rotator. The plasma acts to cancel the parallel electric fields and moves along the magnetic field tracing a cone around the rotation axis.

#### 5. Conclusion

Our findings can be summarized as follows: pulsar magnetospheres *can* be filled by surface emission with the help of a 3D diocotron instability; simple models of oblique rotators *can* form relativistic winds; such winds have variation in properties with latitude, with equatorial flow being formed at the edge of the closed magnetosphere, and polar cap flow probably contributing in the direction of rotational axis. Much work remains to be done to quantify these qualitative results and to understand the dependence of the wind geometry, acceleration

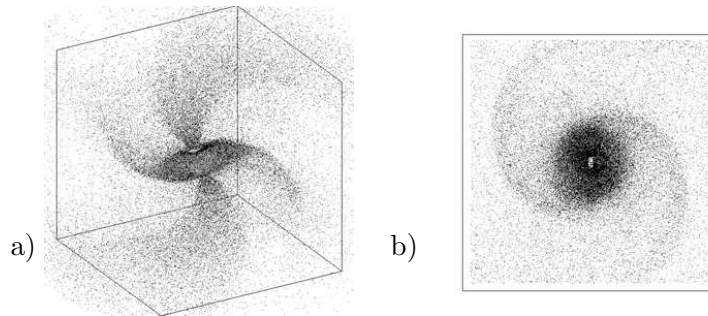


Figure 2. a) Plasma distribution in a  $20^\circ$  inclined rotator. Shown are the closed magnetosphere, equatorial outflow, and the polar flux tube; b) View of the equatorial plane illustrating the spiral wave formation. For the movie version of this figure please see <http://astron.berkeley.edu/~anatoly/magnetosphere.html>

and collimation on the parameter regime. While tempting, it would be too early to suggest that the equatorial and polar outflows observed in these simulations are directly responsible for features in the Crab and Vela. The models described in this paper are self-consistent in the sense that the field and particle equations are solved simultaneously and correctly. However, the models are not currently self-consistent with respect to the boundary conditions: the induced quadrupole electric fields are those from a rotating magnetized *conductor*, but as far as the plasma is concerned the central body is a *dielectric* with  $\epsilon = 1$ . Both KPM1, KPM2 and SMT simulations suffer from the same problem as well. This restriction does not allow currents to close inside the star, and instead leads to charging of the central body at the locations where the current is extracted and returned. Consequently, any current flow in the system is temporary. We are currently working on overcoming this deficiency. The more realistic models with true central conductors will be described in detail in an upcoming publication.

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